N95-13898

SUPERSYMMETRY AND RADIAL SQUEEZED STATES FOR RYDBERG WAVE PACKETS

Robert Bluhm[a] and V. Alan Kostelecký[b1]

[a]*Physics Department, Colby College*
Waterville, ME 04901, U.S.A.

[b]*Physics Department, Indiana University*
Bloomington, IN 47405, U.S.A.

Atomic supersymmetry provides an analytical effective-potential model useful for describing certain aspects of Rydberg atoms. Experiments have recently demonstrated the existence of Rydberg wave packets localized in the radial coordinates with p-state angular distribution. This talk shows how atomic supersymmetry can be used to treat radial Rydberg wave packets via a particular analytical type of squeezed state, called a radial squeezed state.

1. Introduction

Irradiation of a Rydberg atom with a short laser pulse can produce a radially localized wave packet with a p-state angular distribution. The time evolution of such a state initially exhibits some attributes of the classical radial motion, including the Kepler period [1, 2, 3, 4]. After several radial oscillations, the packet disperses. At various later times, the quantum wave recombines into single- or multiple-component packets called revivals [5, 6, 7, 8, 9].

The localization of the initial packet suggests a theoretical description via a coherent or squeezed state [10]. However, a direct approach meets technical obstacles [11] or generates quantum packets that do not describe the characteristics of radial p-state excitations of Rydberg atoms in the absence of external fields.

This talk provides a summary of our recently developed framework for the analytical study of Rydberg wave packets [12, 13]. Our approach incorporates non-hydrogenic aspects of the packets using atomic supersymmetry [14, 15]. This provides an effective central potential along with analytical wavefunctions $R_{n^*l^*}$ and exact Rydberg eigenenergies E_{n^*} for a Rydberg electron, expressed in terms of shifted quantum numbers n^* and l^*. Recent summaries of the methods and results of atomic supersymmetry and references to the literature can be found in Ref. [16].

[1]Speaker

We use the analytical wavefunctions of atomic supersymmetry to construct a family of analytical squeezed states, called radial squeezed states, that form representations of radial Rydberg wave packets. The procedure begins by mapping the classical physics associated with the effective potential into the form of a harmonic oscillator. After the conversion to quantum physics, squeezed states can be derived for the resulting uncertainty relation. The method is based on an extension of the approach of Ref. [17] for cirular states of hydrogen. The procedure is outlined in sections 2 and 3 below. The time evolution of the resulting radial squeezed states is briefly discussed in section 4. In what follows, we use atomic units with $\hbar = e = m_e = 1$.

The reader is referred to Refs. [12, 13] for more details about the subjects presented in this talk and for more references to the background material.

2. Classical Physics

The classical theory corresponding to atomic supersymmetry uses a central potential that leads to the effective one-particle radial hamiltonian

$$H^* \equiv \tfrac{1}{2}p_r^2 + \frac{l^{*2}}{2r^2} - \frac{1}{r} = E^* \equiv -\frac{1}{2n^{*2}} \quad , \tag{1}$$

where $p_r = \dot{r}$ is the radial momentum. The classical continuous variable l^* is a shifted value of the classical angular momentum l, arising from the incorporation of the effects of the central potential. At the quantum level, l^{*2} becomes the quantized quantity $l^*(l^*+1)$ of atomic supersymmetry. The energy E^* has for convenience been expressed in terms of a continuous classical variable n^*, which at the level of quantum physics converts to the quantized, shifted principal quantum number of atomic supersymmetry.

When E^* is negative, the particle is bound and oscillates between outer and inner apsidal points, r_1 and r_2, say. The classical orbital period $T^*_{\rm cl}$, which is the time taken to move from r_1 to r_2 and back, is given via Eq. (1) as

$$T^*_{\rm cl} = 2\pi n^{*3} \tag{2}$$

The classical orbit can be shown from Eq. (1) to be a precessing ellipse, obeying the equation

$$\frac{1}{r} = \frac{1}{l^{*2}}\left(1 + e\cos[f(\theta - \theta_0)]\right) \tag{3}$$

Here, $e = \sqrt{1 - l^{*2}/n^{*2}}$, θ_0 is a constant of the integration, and $f = l^*/l$. For the radial Rydberg wave packets we discuss below, l^*/n^* is small and so the corresponding classical orbits are highly elliptical. For simplicity in what follows, we impose $f\theta_0 = \frac{\pi}{2}$.

Direct efforts to obtain coherent or squeezed states based on this model face technical obstacles. Instead, we map the theory into a harmonic-oscillator formalism. We convert to a new set of classical variables, R and P, chosen to have sinusoidal variation with the angle θ and given by

$$R \equiv \frac{1}{r} - \frac{1}{l^{*2}} = \frac{e}{l^{*2}} \sin f\theta \quad , \qquad P \equiv -\frac{r^2}{f}\dot{R} = -\frac{el}{l^{*2}} \cos f\theta \quad . \tag{4}$$

In terms of these variables, the classical equations of motion become

$$\dot{R} = -\frac{f}{r^2} P \quad , \qquad \dot{P} = \frac{l^2 f}{r^2} R \quad . \tag{5}$$

Equation (1) then takes the form of the energy equation for a simple harmonic oscillator of frequency l and energy $e^2/2f^2$:

$$\tfrac{1}{2}P^2 + \tfrac{1}{2}l^2R^2 = \frac{e^2}{2f^2} \quad . \tag{6}$$

3. Quantum Physics

At the quantum level, the new classical variables R, P become quantum operators given by

$$R = \frac{1}{r} - \frac{1}{l^*(l^*+1)} \quad , \qquad P = \frac{p_r}{f} = -\frac{i}{f}(\partial_r + \frac{1}{r}) \quad , \tag{7}$$

obeying the commutation relation

$$[R, P] = -\frac{i}{f}\frac{1}{r^2} \quad . \tag{8}$$

The uncertainty product $\Delta R \Delta P$ is

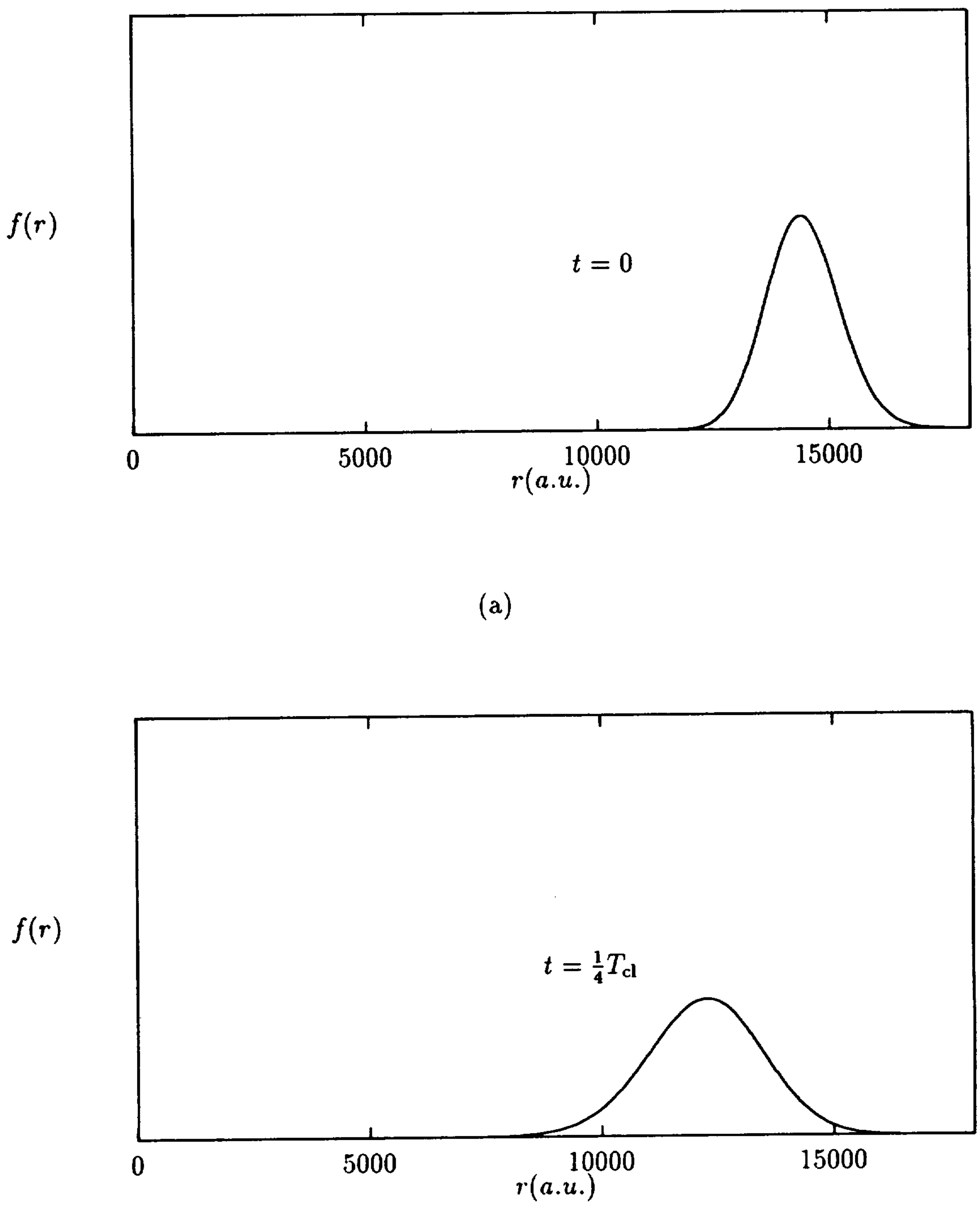
$$\Delta R \Delta P \geq \frac{1}{2f}\langle \frac{1}{r^2} \rangle \quad . \tag{9}$$

At a given time, any minimum-uncertainty wavefunctions must therefore obey the equation

$$(R - \langle R \rangle)\psi = iA(P - \langle P \rangle)\psi \quad , \tag{10}$$

where the real constant A is given by

$$A = \frac{2f(\Delta R)^2}{\langle \frac{1}{r^2} \rangle} = \frac{\Delta R}{\Delta P} \quad . \tag{11}$$



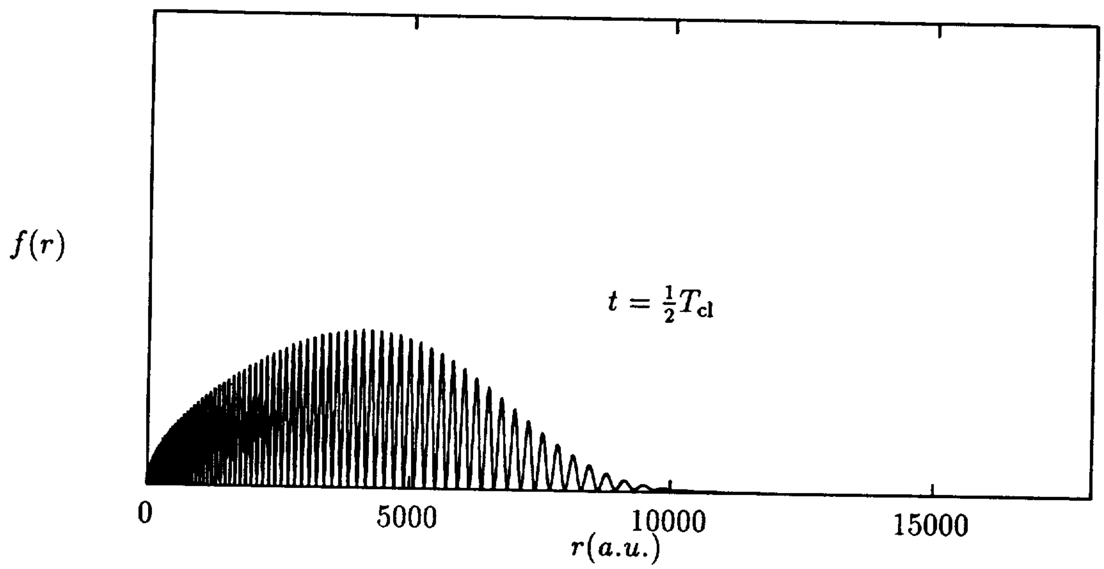
f(r)
t = 0
0
5000
10000
15000
r(a.u.)

(a)

f(r)
$t = \frac{1}{4}T_{cl}$
0
5000
10000
15000
r(a.u.)

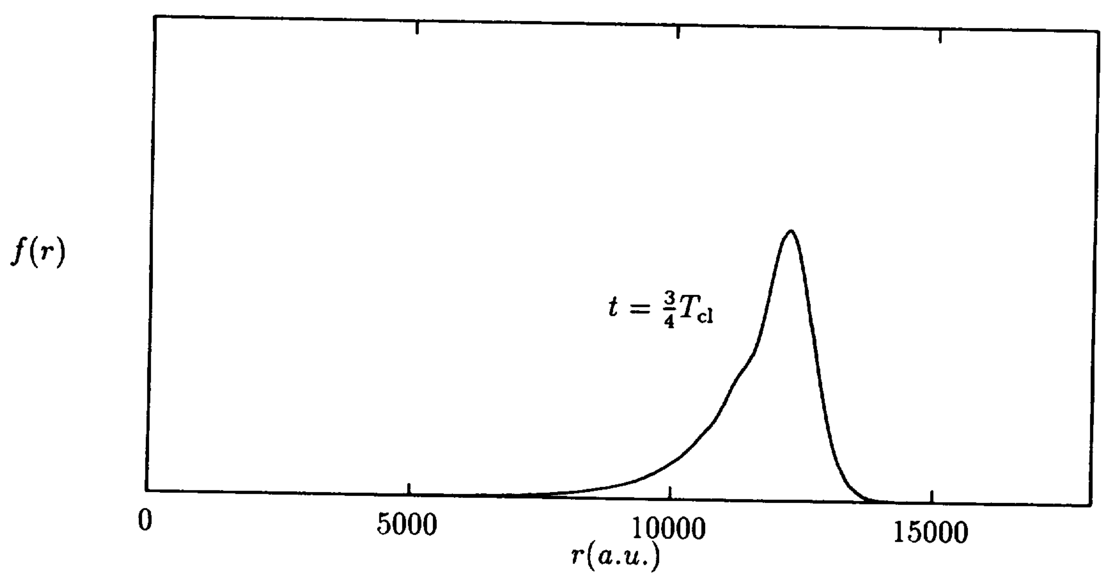
(b)

Figure 1



f(r)
$t = \frac{1}{2}T_{cl}$
0
5000
10000
15000
r(a.u.)

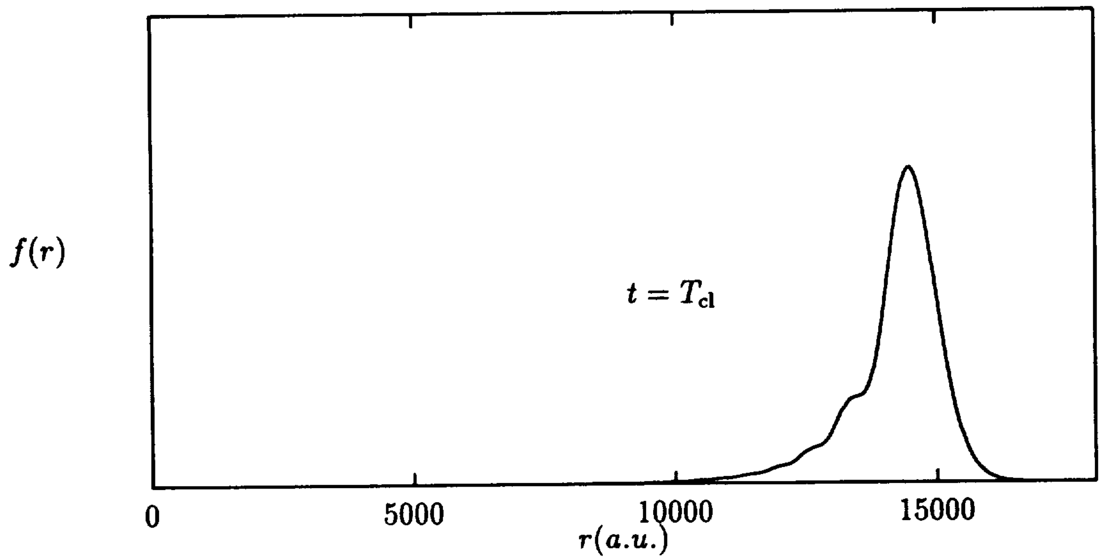
(c)



f(r)
$t = \frac{3}{4}T_{cl}$
0
5000
10000
15000
r(a.u.)

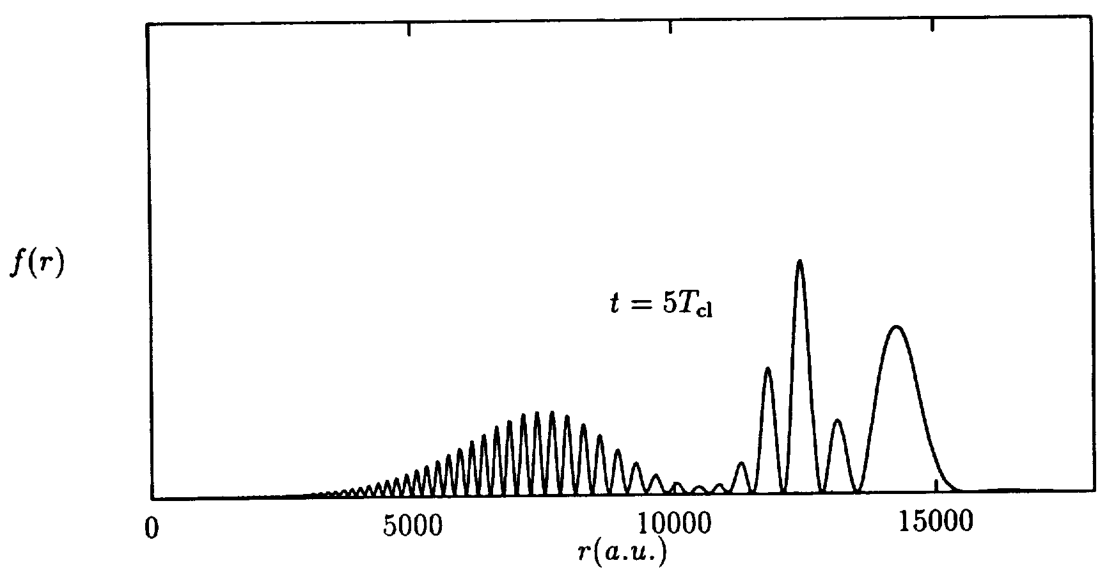
(d)

Figure 1



$f(r)$
$t = T_{\rm cl}$
0
5000
10000
15000
$r(a.u.)$

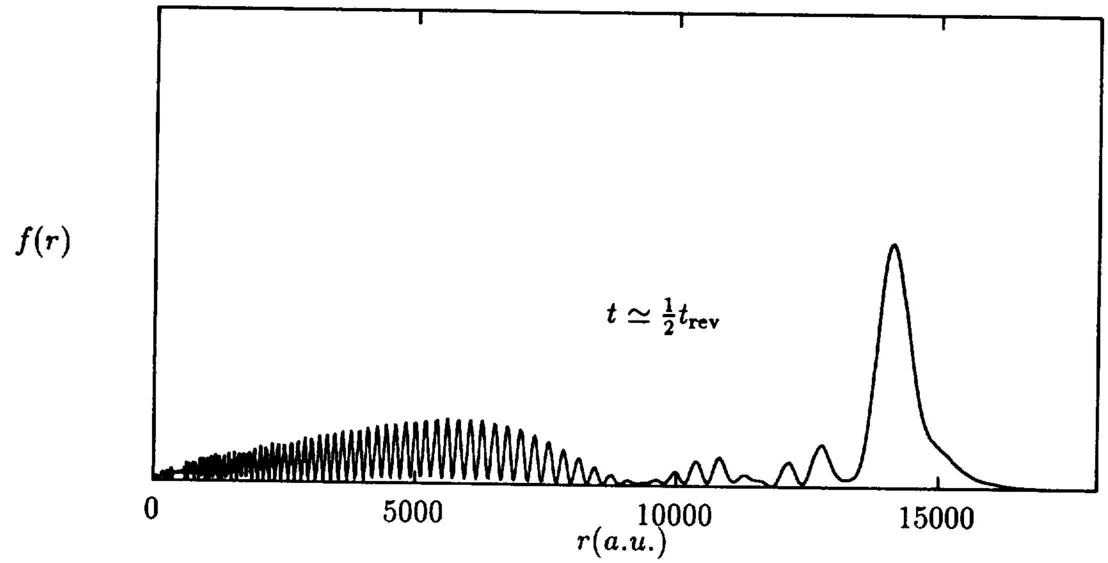
(e)



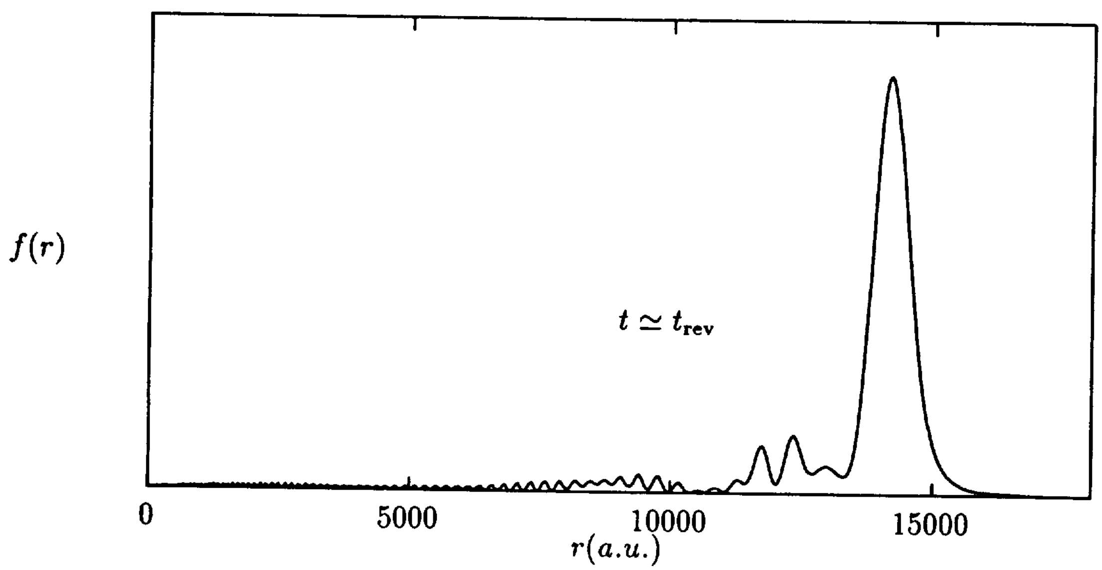
$f(r)$
$t = 5T_{\rm cl}$
0
5000
10000
15000
$r(a.u.)$

(f)

Figure 1


f(r)
t ≃ ½t_rev
0
5000
10000
15000
r(a.u.)

(g)


f(r)
t ≃ t_rev
0
5000
10000
15000
r(a.u.)

(h)

Figure 1

At later times, fractional and full revivals appear. The fractional revival consisting of two separate peaks, which appears at $t_r^* = t_{\rm rev}^*/2 \simeq 1.3$ nsec., is shown in Figure 1g, while the full revival appearing at $t_{\rm rev}^* \simeq 2.6$ nsec. is shown in Figure 1h. These revivals exhibit all the features expected from the classical analysis, including, for example, the expected values for the wavefunction periodicity.

Along with the additional analysis contained in Refs. [12, 13], these results indicate that radial squeezed states are useful models for radial Rydberg wave packets.

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